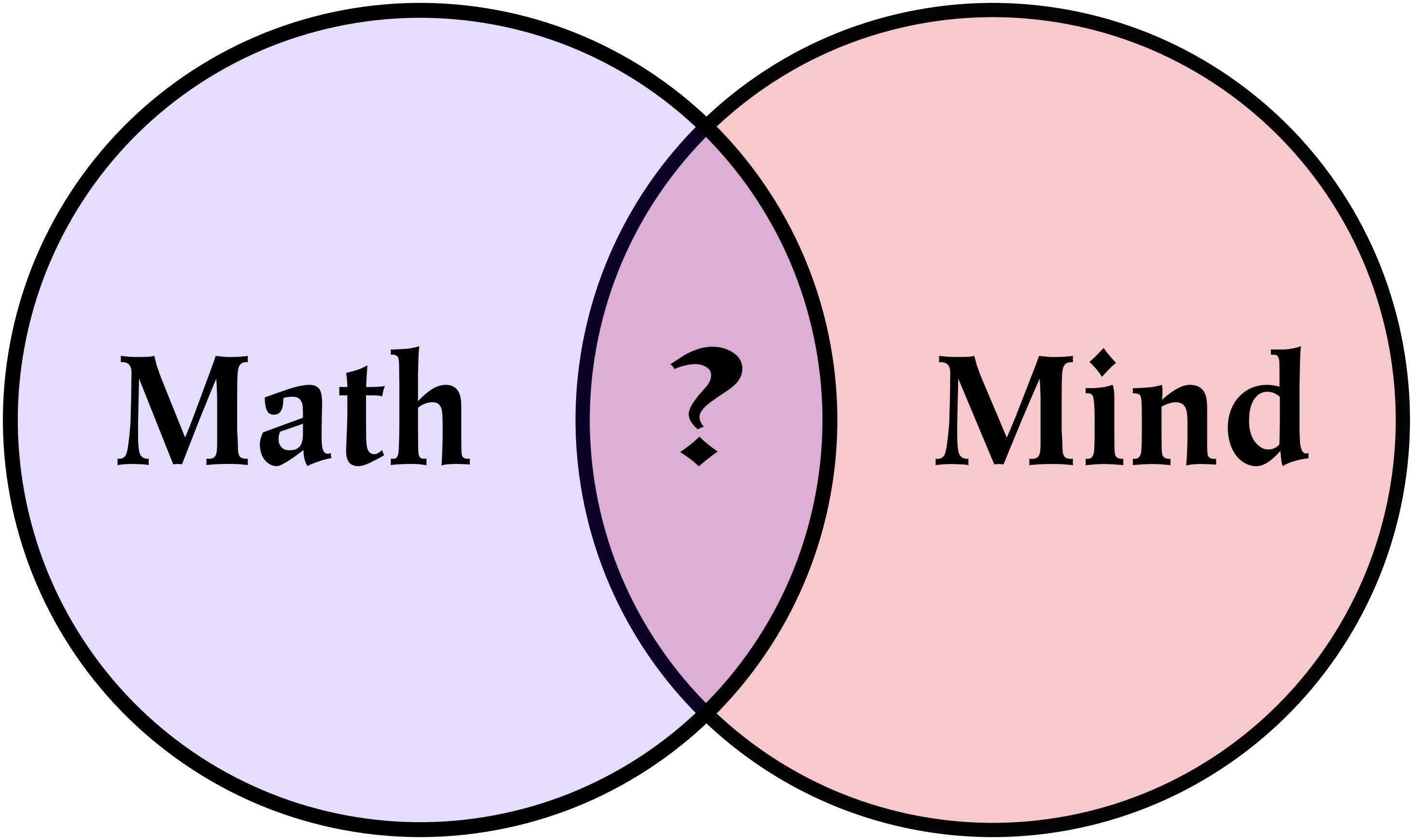
Math
?
Mind

Mathematics of Enlightenment

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1: Overview

The intent of this paper is to establish a formal psychological model with which to understand enlightenment. This formal model borrows heavily from two mathematical disciplines: set theory and mereotopology. These two fields are chosen because they correspond to a key distinction in many models of enlightenment: the distinction between conceptual and non-conceptual mind.

After establishing several features of this formal psychological model, selflessness and meditation are briefly discussed from within this framework. This discussion borrows heavily from Buddhist philosophy, and uses Tibetan and Gelukba terminology, but it should be compatible with a large number of meditative traditions. It is hoped that the use of this formal psychological model to discuss enlightenment will result in greater clarity.

Mathematics	**Psychology**
Set Theory (symbolic logic, algebra)	Conceptual Mind
Topology (mereology, geometry)	Non-conceptual mind

2: Enlightened Mind

In order to formulate a mathematical basis for enlightenment, we adopt the following (generic) description of enlightenment from an external perspective (i.e. we do not address the subjective experience of enlightenment):

> *Enlightenment consists of the development of selflessness*

Selflessness is understood to have both an intellectual and emotional component:

- The wisdom of selflessness entails non-conceptual knowing, which is often occluded by conceptual or thinking mind.

- The emotion of selflessness entails unconditional love, which is occluded by attachment and aversion to particular objects.

This paper deals with concepts rather than emotions, not because they are more important, but because they can be described more easily. However, since the use of particular concepts is closely tied to our emotions with respect to those concepts, the role of emotions cannot be ignored in practice.

2.1: Non-conceptual mind

Non-conceptual mind is a *knowing* mind. When we meditate, we are encouraged to get in touch with this mind, which is obscured due to our *kleshas* (or karma). These kleshas prevent us from seeing things in an objective way; they induce us to see things from a self-centered perspective. This isolation of a particular point of view prevents other points of view from being seen.

Although the non-conceptual mind knows things, it does not know them in isolation from one another. Because it knows things, some people might prefer the term multi-conceptual mind. In either case, it is a non-karmic (or unconditioned) mind, which is similar to intuition (although it is not limited to operating in brief flashes).

The picture below depicts a cup. As seen by non-conceptual mind, it is not merely a cup, but a green table, some amount of coffee, a handle, and any number of other things. Non-conceptual mind always sees things in their context, and in relationship to other things.

Figure 1: Perception of a Cup

2.2: Conceptual Mind

According to Buddhist tradition, a “real thing” (or *svalaksana*) is defined to be:

- **Singular** (*eka*): If a thing is singular, then it is a whole which cannot be broken into parts.
- **Independent** (self-existent): If a thing is independent of all other things, then it is defined entirely in terms of its contents (as opposed to being defined in terms of other things).
- **Permanent** (unchanging): If a thing is permanent, then it endures through time.

Concepts corresponding to these things are of two types: meaning generalities (*don spyi*), which are formed through the negation of their (perceptual) complement, and term generalities (*sgra spyi*), which are formed through the use of other concepts. Concepts isolate the object under consideration: they remove that object from its context. This process (which is a summary of Apoha theory) is depicted graphically by image of the previous section and the two images below.

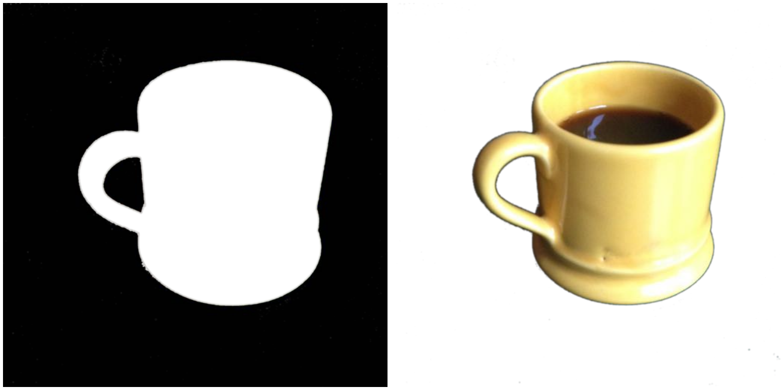

Figure 2: "Not-Cup" and "Negation of Not-Cup"

3: Mathematics

The fields of mathematics that are most appropriate to the distinction between non-conceptual and conceptual mind are mereotopology and set theory, respectively.

Mereology (literally the study of parts) is a nominalistic version of set theory [Lesniewski]. It is combined with topology (the study of places) to form *mereotopology*, which analyzes regions in terms of parts, wholes, and boundaries [Whitehead, Caseti and Varzi]. Mereotopology can begin with an undivided whole, which makes it a non-reductionistic alternative to "pointy" versions of topology (that posit an infinite set of infinitesimal (and indivisible) points).

Set theory is mathematical formalism for collections of things that was developed by the German mathematician Ernst Zermelo. The things which are collected are called *elements*, and the collections themselves are called *sets*. Set theory is famous for providing a common framework for most, if not all, of mathematics. Central to the notion of set theory is that it builds on itself: initially, there are no sets, and only elements. Next, we may compose sets by collecting the elements. After that, we may compose sets by collecting the elements and the sets that have just been created, which leads to a hierarchy of set composition.

3.1: Mereotopology

Mereotopology derives topological notions, such as enclosure, connection, and overlap, from the notion of parthood (i.e. the *part* function). The part function plays a role similar to the element-of operator in set theory, but there are significant differences. First of all, parts are created from a larger whole, whereas sets are constructed from elements. Further, parts do not have a unique decomposition into their subparts: they are continuous (sets, on the other hand, have an enumerable number of subsets). Parts do not rely on points, as the example below demonstrates.

```
Universe
cup = part(Universe)
not(cup) = Universe - cup
handle = part(cup)
```

Figure 3: Composition of Parts

Parts can be depicted hierarchically, which creates *meronomies*. An example of a meronomy is shown below (the diamond arrow heads indicate the "has-a" or "is a part of" relationship).

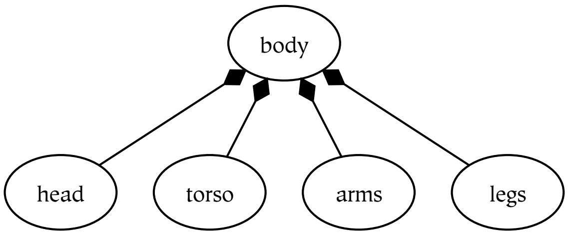

Figure 4: A Meronomy

3.2: Set Theory

Sets are singular things that are composed of other sets, and ultimately of *elements*. They are defined by their contents: in other words, they exist in virtue of what they contain (or "from their own side").

Sets have a unique decomposition into their constituent sets (or subsets): {{a,b}, c} is not the same as {{a}, {b,c}}. Similarly, they are not transitive: the subset of a set is not (necessarily) a subset of the original set. In other words, sets do not behave like parts (the part of a part of a whole is necessarily a part of that whole).

Rank0 sets: {}
Rank1 sets: {{}}, ...
Rank2 sets: {{{}}}, {{{}},{}}, ...

Figure 5: Composition of Sets

Sets compose to create hierarchies: the set {all things} may be defined as the composition of the sets {animal things}, {vegetable things}, and {mineral things}. In this example, the set {all things} has exactly three members. An abstract version of this hierarchy is depicted below as a taxonomy (the empty arrow heads indicate the "is-a" or "type of" relationship).

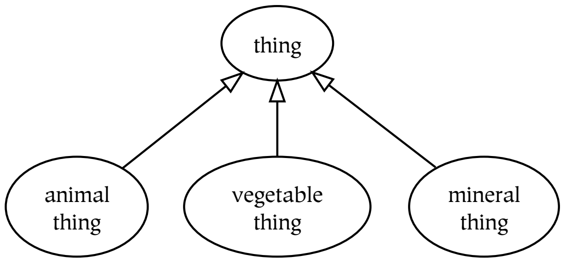

Figure 6: A Taxonomy

4: Results

The first bridge that we make between mathematics and psychology is that between space and mind. The spatial regions that form the objects of thought are parts of a larger (unifying) multidimensional space. The mathematical model used to specify how those parts are created from that larger whole is mereotopology.

The second bridge that we make between mathematics and psychology is that between sets and concepts. The elements of this application of set theory are taken to be the spatial regions identified by attention (or mereology), and the sets that contain these elements are equated with the concepts of conceptual mind.

After establishing these connections in the next two sections, we briefly discuss two ideas central to enlightenment from both perspectives: selflessness and meditation.

Objects (O):	cup, not-cup
Concepts (C) of O:	{cup}, {not-cup}, {}, {cup,not-cup}
C of C of O:	{{cup}}, {{cup}, {not-cup}}}, ...
C of C of C of O:	{{{cup}}}, ...

Figure 7: Sets of Parts (using 'cup' and 'not-cup' as elements)

4.1: Mind and Space

Mereotopology	Non-conceptual mind
Space	Mind
Points	Real (according to Sautrantika)
Regions	As real as points (according to Madhyamika)

We make the assumption that our concepts derive from our experience, so it is important to clarify how they do so (i.e. if concepts correspond to sets, what are the elements?). For example, does a concept refer to a single "real" object or multiple "real" objects? These and related questions have received a lot of attention historically (see the references section for an overview). Here, we adopt the convention that all of our experience takes place within (multidimensional) space, and that the objects within that space are *nominal*: in other words, a given object exists *iff* its name refers to a non-empty region.

Sets group or unite things, so they are a sufficient basis for mathematics if there is nothing smaller than a point. From both Buddhist and psychological perspectives, however, perceptual objects are identified both in terms of their parts and the larger wholes of which they are parts. For this reason, we adopt the view that points (or atoms, used in its original sense) are no more real than larger-scale objects, and seek to establish objects in a non-reductionistic (or holistic) way. Psychologically, this mechanism corresponds to (perceptual) attention, which creates a distinction between figure and ground. Mathematically, this mechanism corresponds to mereotopology, which creates parts out of a larger space.

4.2: Concepts and Sets

Set Theory	Conceptual Mind
Sets	Concepts
$Rank_0$ sets	Meaning generalities (don spyi)
$Rank_N$ sets	Term generalities (sgra spyi)

The identification of sets with concepts seems intuitive: both sets and concepts are single things that contain a multiplicity. Establishing this bridge brings interesting insights to both domains.

One way in which set theory refines our understanding of conceptuality is by introducing the notion of *rank*. The rank of a set indicates how far that set is abstracted from the ground of its elements. In more Buddhist (and particularly Gelukba) terms, set theory makes a significant distinction between meaning generalities and multiple levels of term generalities. The rank of a concept indicates how far we have strayed from the direct perception of reality. Note, however, that it is not the concept itself which determines its rank. For someone who has never experienced ice directly, the concept "ice" will necessarily have a rank greater than zero, but "ice" for someone who has known ice directly may be a $rank_0$ concept.

Further, the utilization of high-rank concepts seems to be a likely cause for the (erroneous) notion that things are both independent and permanent. In particular, because high-rank concepts (or term generalities) are increasingly abstract, their (spatial) context becomes increasingly less distant (measured in terms of intermediary concepts).

4.3: Selflessness

The Buddhist term “selflessness” (or *emptiness*) entails that things do not exist in the way that we commonly think they do. It is also used to indicate that a person or an object has a specific type of emptiness which is not realized: in other words, realizing the emptiness of one thing does not entail realizing the emptiness of another. While the generic term *emptiness* conveys the presence or lack of a larger context, the *emptiness of a particular object* may be expressed as the (semantic) complement of that object. Mathematically, we define the emptiness of a thing as the complement (or the negation) of that thing. Therefore, two notions of complement must be elucidated: mereological complement and set complement.

With respect to mereological complement, if the emptiness of a part is the complement of that part, then perceiving the emptiness of an object entails *not* narrowing attention to just that part. This narrowing of attention typically happens when an object is conceived by the mind (this is clarified in Apoha theory, which holds that an object presents itself to our mind in virtue of the negation of its complement).

With respect to the set-theoretic complement, if the emptiness of a concept is the complement of that concept, we must define the larger context under which that complement may be formed. There seem to be two clear alternatives for doing so: we may define the emptiness of a concept "cup" as the concept "not-cup". We might also define the emptiness of cup as the spatial complement of the region referenced by cup. Since the ultimate aim of the perception of emptiness involves non-conceptual mind, the later definition seems preferable.

4.4: Meditation

Knowledge involves both the correct identification of objects and understanding the relations between those objects. Although conceptual mind can perform both of these tasks, it is a biased version of non-conceptual mind, that forgets certain details in order to focus on others.

Several forms of Buddhist practice (such as the noting practice outlined in the Satipatthana sutra, or various kinds of mindfulness and awareness practice) entail the cessation of the proliferation of thought. In terms of the mathematical formulation developed here, thought requires concepts of a high rank. With respect to the suggested practice, we may ask if that practice suggests the formation of sets of rank_0 exclusively, or is an injunction against the formation of sets at all?

Explicit verbal labels for the rank_0 concepts formed during noting practice are probably unnecessary: although the nun knows what she is seeing, she does not mentally repeat the name of the thing which she is seeing. Concepts of any higher rank (i.e. term generalities) are prohibited, since they do not directly know their object (they directly know other concepts, which is what makes them abstract). The question of whether knowledge is possible without the formation of even rank_0 concepts will be left to future research.

5: References

Knowledge and Liberation: Tibetan Buddhist Epistemology in Support of Transformative Religious Experience, Anne C. Klein

Foundations of Dharmakirti's Philosophy, John D. Dunne

Parts and Places: The Structures of Spatial Representation, Roberto Caseti and Achille Varzi

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Conceptual Spaces: The Geometry of Thought, Peter Gärdenfors

Where Mathematics Comes From: How The Embodied Mind Brings Mathematics Into Being, George Lakoff, Rafael Nuñez

Cognitive Set Theory, Alec Rogers

Several pages that were removed from this presentation are available online:

http://cognitivesettheory.com/MathematicsOfEnlightenment

6: Conclusion and Future Directions

By combining mathematics and psychology, we obtain a formal foundation for talking about the mind and its operations. This foundation:

- Assumes that the basis of mind can be represented by a spatial logic. We assume that the mind is spatial, and look to mathematics for a suitable spatial formalism.
- Provides a solid foundation for talking about the mind and its operations. For example, saying that "Mindfulness does not consist of forming the set of objects of which one wishes to be mindful" has a clear mathematical formulation.
- Yields a meaningful application of mathematics. Using psychological motivations for mathematical principals is arguably better than "the fewest number of axioms wins".
- Does not provide, or attempt to provide, the biological implementation of mind. Although there is a popular trend to explain the mind in this reductionistic way, the need for large-scale models will not go away when we understand the operation of the mind on a small scale.

Hopefully this formalism will have positive practical consequences: may it benefit all beings.